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DEFENCE RESEARCH CENTRE SALISBURY  
SOUTH AUSTRALIA

**TECHNICAL REPORT**

WSRL-0196-TR

**MAXIMUM ENTROPY ESTIMATES OF THE WAVENUMBER POWER SPECTRUM  
OF ACOUSTIC DATA FROM A LINEAR ARRAY OF EQUISPACED SENSORS**

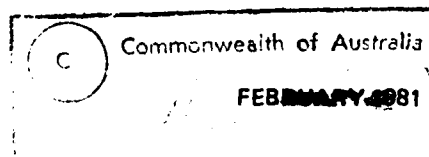
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TECHNICAL REPORT

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MAXIMUM ENTROPY ESTIMATES OF THE WAVENUMBER POWER SPECTRUM  
OF ACOUSTIC DATA FROM A LINEAR ARRAY OF EQUISPACED SENSORS

D.A. Gray

S U M M A R Y

The maximum entropy method may be used to estimate the wavenumber power spectrum of data from a linear array of equispaced sensors by extrapolating a spatial covariance function. The estimation of this spatial covariance function from real data is discussed. An example is given which compares the Fourier and maximum entropy estimates of the wavenumber power spectrum of sonar data from a linear array of equispaced receivers at a number of frequencies. The sensitivity of the maximum entropy method to the length of the estimated covariance function is also illustrated.



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The maximum entropy method may be used to estimate the wavenumber power spectrum of data from a linear array of equispaced sensors by extrapolating a spatial covariance function. The estimation of this spatial covariance function from real data is discussed. An example is given which compares the Fourier and maximum entropy estimates of the wavenumber power spectrum of sonar data from a linear array of equispaced receivers at a number of frequencies. The sensitivity of the maximum entropy method to the length of the estimated covariance function is also illustrated.

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## 1. INTRODUCTION

The maximum entropy method (MEM) has been successfully used (ref.1,3) to estimate the power spectrum of severely-truncated time series. The enhanced resolution of the MEM power spectral estimates are due to the estimation of a set of linear prediction filter coefficients which can be used to extrapolate either the data or the covariance function of the data outside a finite observation interval.

In frequency domain beamforming the estimation of the wavenumber power spectrum using data from a linear array of equispaced sensors is well-known to be analogous to the estimation of the power spectrum of a time series. As shown in Section 2 the wavenumber power spectrum at each frequency of interest is the Fourier transform of a complex spatial covariance function which can readily be derived from the cross-power spectral matrix. For an array of limited aperture MEM can then be used to extrapolate this spatial covariance function independently at each frequency. It should be pointed out that the estimates of the frequency wavenumber spectrum discussed in this report differ from the two-dimensional MEM recently proposed by Woods (ref.4). In the application considered here the extrapolation in the time domain is unnecessary since increased frequency resolution can easily be obtained by increasing the length of the time data sequence from each sensor.

In Section 4 the estimation of the spatial covariance function of data from a linear array of equispaced sensors is discussed and the effect on the MEM of biases in estimates of the covariance function due to windowing is shown.

Finally in Section 5 the methods discussed in the previous sections are used to estimate the maximum entropy frequency wavenumber power spectrum of sonar data from a linear array of equispaced sensors. For comparison the Fourier estimates of the frequency wavenumber power spectrum were calculated and are also presented. The sensitivity of the MEM to the length of the estimated spatial covariance function is illustrated by means of an example.

This work is part of a continuing R&D programme in signal processing for underwater acoustics and has been carried out under task DST 79/069.

## 2. THE FREQUENCY WAVENUMBER POWER SPECTRUM

In beamforming, the power spectrum of noise incident upon the array is often estimated as a function of frequency and the angular co-ordinates. Because of the rotational symmetry of a linear array only the total power density incident on the array from a cone defined by an angle  $\theta$  relative to the axis of the array can be measured. Thus the noise field can be estimated only as a function of frequency and the angle  $\theta$ . An alternative approach (discussed in more detail in reference 5) is to use the single-dimension co-ordinate wavenumber\*  $k$ , defined by

$$k = \sin \theta / \lambda$$

instead of  $\theta$ . The rationale for using the frequency wavenumber approach has been discussed in reference 5 and is adopted in this report.

Denoting the  $\ell$ -th sampled output of the  $j$ -th receiver by  $x_j(\ell\tau_o)$  where  $\tau_o$  is the sampling interval, it follows that if  $A(f,k)$  is defined by

\*  $\sin \theta / \lambda$  is strictly  $k_x$ , the component of the wavevector along the axis of array. Since only one dimension is considered, the  $x$  subscript is dropped and  $k$  is referred to as wavenumber.



$$A(f,k) = \lim_{M \rightarrow \infty} \lim_{N \rightarrow \infty} \frac{1}{M+1} \frac{1}{N+1} \sum_{j=-M/2}^{M/2} \sum_{\ell=-N/2}^{N/2} x_j(\ell \tau_0) e^{-2\pi i f \ell \tau_0} e^{-2\pi i k j d}, \quad (1)$$

where  $d$  is the separation of adjacent receivers, then the two-dimensional power spectral function is given by

$$S(f,k) = \langle A(f,k) A^*(f,k) \rangle,$$

where  $\langle \rangle$  denotes ensemble averaging.

Defining

$$x_j(f) = \lim_{N \rightarrow \infty} \frac{1}{N+1} \sum_{\ell=-N/2}^{N/2} x_j(\ell \tau_0) e^{-2\pi i f \ell \tau_0}$$

and

$$r_j(f) = \lim_{M \rightarrow \infty} \frac{1}{M+1} \sum_{i=-M/2+j}^{M/2-j} \langle x_i(f) x_{j+i}^*(f) \rangle \quad (2)$$

then it can easily be shown that

$$S(f,k) = \lim_{M \rightarrow \infty} \frac{1}{M+1} \sum_{j=-M/2}^{M/2} r_j(f) e^{-2\pi i j k d}.$$

By analogy with time series analysis it is natural to term  $r_j(f)$  the spatial covariance function since its infinite Fourier transform is the wavenumber spectrum. One important difference is that  $r_j(f)$  is in general complex whilst for time series analysis the covariance function in general is real.

For an array of finite extent  $r_j(f)$  can only be estimated for  $|j| \leq M'$  where  $M'd$  is the aperture of the array. Hence a natural estimator is obtained by replacing the infinite sum in equation (1) with a finite one. For an array with a finite number of receivers and for a finite number of sampled data points the Fourier estimate  $\hat{S}_B(f,k)$  is defined to be

$$\hat{S}_B(f,k) = \langle \hat{A}_B(f,k) \hat{A}_B^*(f,k) \rangle$$

where

$$\hat{A}_B(f, k) = \frac{1}{M+1} \sum_{j=-M/2}^{M/2} \hat{x}_j(f) e^{-2\pi i k j d}$$

and where

$$\hat{x}_j(f) = \frac{1}{N+1} \sum_{l=-N/2}^{N/2} x_j(l\tau_o) e^{-2\pi i f l \tau_o} \quad (3)$$

The subscript B has been used to denote that, in forming this two-dimensional periodogram, a Bartlett window has been implicitly applied to the spatial covariance because of the presence of the  $\frac{1}{M+1}$  term in the summation.

### 3. MAXIMUM ENTROPY ESTIMATES OF THE WAVENUMBER SPECTRUM

At each frequency the MEM may be used to extend the spatial covariance function derived from a finite aperture array. As discussed in the introduction each frequency is treated separately on the assumption that time series outputs from all receivers can be made sufficiently long to achieve any desired frequency resolution. Thus the method of Woods for estimating two-dimensional maximum entropy spectrum need not be used and a simple extension of the method of Edwards and Fitelson(ref.6) into the complex domain can be used. The method is outlined in this section for the sake of completeness.

The entropy at any given frequency  $f$  for a process with a wavenumber spectral density  $S(f, k)$  may be defined as

$$H(f) = \int_K \log S(f, k) dk \quad (4)$$

where the integration runs over the values of  $k$  for which  $S(f, k)$  is non zero. The maximum entropy estimates  $\hat{S}_{ME}(f, k)$  of  $S(f, k)$  are derived by maximising (4) subject to the constraint that the inverse Fourier transform of  $\hat{S}_{ME}(f, k)$  is equal to  $\hat{r}_j(f)$ . (A more recent derivation of the method by Newman(ref.7) does not require equality but a close approximation to the observed  $\hat{r}_j(f)$ 's. This method is not considered in this report).

The problem of maximizing (4) subject to the constraint

$$\int_{-k_o}^{k_o} \hat{S}_{ME}(f, k) e^{-2\pi i k j d} dk = \hat{r}_j(f)$$

for  $j = 0, \pm 1, \dots, \pm(N-1)$

and where

$$\hat{r}_j^*(f) = \hat{r}_{-j}(f)$$

can readily be solved by introducing Lagrange multipliers à la Edwards and Fitelson. The solution is

$$S(f, k) = \frac{P_N}{\left| \sum_{j=0}^{N-1} a_j e^{-2\pi i k j d} \right|^2} \quad (5)$$

where  $P_N$  and the  $a_j$  ( $a_0 = 1$ ) are solutions of the complex equation

$$\begin{pmatrix} r_0 & r_1 & \cdot & \cdot & \cdot & \cdot & r_{N-1} \\ r_{-1} & r_0 & & & & & \cdot \\ \cdot & & \cdot & & & & \cdot \\ \cdot & & & \cdot & & & \cdot \\ \cdot & & & & \cdot & & \cdot \\ \cdot & & & & & \cdot & \cdot \\ \cdot & & & & & & r_1 \\ r_{-N+1} & \cdot & \cdot & \cdot & \cdot & r_{-1} & r_0 \end{pmatrix} \begin{pmatrix} 1 \\ a_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ a_{N-1} \end{pmatrix} = \begin{pmatrix} P_N \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 0 \end{pmatrix} \quad (6)$$

It is worth commenting that  $P_N$  can be related to final error of a prediction filter which uses  $N-1$  previous values to estimate one lag ahead. A wealth of papers exists on efficient algorithms for solving the Toeplitz equation (6) (see reference 8).

As a final comment reference should be made to an alternative method developed by Burg(ref.2) whereby the prediction filter coefficients (ie the  $a$ 's) can be estimated directly from the data without the need to estimate the covariance function.

#### 4. ESTIMATION OF THE SPATIAL COVARIANCE FUNCTION

The application of the MEM described in the previous section requires that the spatial covariance function at each frequency of interest be estimated. From equation (2) a reasonable estimate of  $r_j(f)$  would appear to be

$$\hat{r}_j(f) = \frac{1}{M+1} \sum_{i=-M/2+j}^{M/2-j} \langle \hat{x}_i(f) \hat{x}_{j+i}^*(f) \rangle_P \quad (7)$$

where  $\langle \rangle_P$  denotes averaging over P independent estimates, ie

$$\langle \hat{x}_i(f) \hat{x}_{j+i}^*(f) \rangle_P = \frac{1}{P} \sum_{k=0}^{P-1} \hat{x}_i^{(k)}(f) \hat{x}_{j+i}^{(k)*}(f). \quad (8)$$

with

$$\langle \hat{x}_i^{(\ell)}(f) \hat{x}_k^{(m)*}(f) \rangle_P = 0 \quad \ell \neq m$$

In general a good approximation to equation (8) can be obtained by segmenting the total time series into P consecutive blocks.

It can easily be shown that the estimate  $\hat{r}_j(f)$  given by equation (7) is biased.

The process of averaging reduces the variance of the estimator  $\hat{r}_j(f)$  but not the bias which is caused by the implicit weighting function  $\frac{1}{M+1}$  (ie the Bartlett weighting in equation (7)). To illustrate the seriousness of this type of bias in  $\hat{r}_j(f)$  consider the example of sine wave in white noise. The time series argument has been chosen here but it can easily be shown that it is trivial to extend the following argument to the spatial domain. Replacing the finite average by the ensemble average, equation (7) reduces to

$$\hat{r}_j(f) = \frac{|M-j|}{M+1} \left\{ \delta_{j0} + a \cos 2\pi f j \tau_0 \right\}$$

where

$$\delta_{j0} = \begin{cases} 1 & j = 0 \\ 0 & j \neq 0 \end{cases}$$

and  $a$  is the signal-to-noise power ratio. A plot of  $\hat{r}_j(f)$  is given in figure 1 where the Bartlett (or triangular) weighting over the interval (0-T) can readily be seen for  $a=10$ . The Fourier spectrum, the maximum entropy extrapolation of the covariance function and the corresponding maximum entropy spectrum are also shown in figure 1.

As can be seen the MEM interprets the weighting on the covariance function as a modulation, extrapolates both the modulation and the carrier and consequently splits the single spectral line into two closely spaced lines beating together. Thus the bias introduced by using equation (7) as an estimator of the covariance function has a radically different (and vastly misleading) effect on the MEM power spectral estimates than it does on the Fourier ones where it permits a

well-known trade-off between main lobe and side lobe responses. It should be emphasized that this is not a weakness of the MEM - given the distorted covariance matrix it made optimum use of the available information. The same argument is directly applicable to the spatial domain where the corresponding effect would be the splitting of a single arrival into two distinct ones.

The technique adopted in this report for removing the bias is to replace equation (7) by

$$\hat{r}_j(f) = \frac{1}{|2M+1-j|} \sum_{-M/2+j}^{M/2-j} < \hat{x}_i(f) \hat{x}_{j+i}^*(f) >_p \quad (9)$$

which is now an unbiased estimator of  $r_j(f)$ . Unfortunately the variance of this estimator of  $r_j(f)$  increases linearly with  $|j|$ . Thus the variance of the estimate of greatest lag will be  $M$  times the variance of the zero lag estimate. An effect of statistical variations in estimates of the  $r_j(f)$  is move the position of poles in the response function of the linear prediction filter coefficients. In particular the effect of noise on poles just inside the unit circle can be to push them outside the unit circle and consequently the method becomes unstable. This results in the maximum entropy extrapolation expanding instead of decaying exponentially. As discussed in the following section this can be avoided by only using  $M'$  (where  $M' \lesssim \frac{M}{4}$ ) of the available lags.

## 5. APPLICATION TO REAL DATA

The method described in the previous sections has been applied to the estimation of the frequency wavenumber power spectrum of acoustic data from a linear array of 32 equispaced hydrophones.

A block diagram of the processing is shown in figure 2. The outputs from each hydrophone were narrowband filtered (by means of the FFT routine) into 40 independent frequency bins. The algorithm is more efficiently implemented by interchanging the order of summation in equation (9), ie the cross-power spectral matrix

$$\hat{x}_i(f) \hat{x}_j^*(f)$$

was estimated at each integration and then was averaged along the diagonals to give an estimate (ie equation (9)) of the spatial covariance function. This estimate was then smoothed by averaging over 300 independent samples of the spatial covariance function. The assumption was made that the frequency wavenumber power spectrum remained stationary over this period.

At each frequency  $N_L$  lags of the spatial covariance function were used to estimate the  $N_L$  complex prediction filter coefficients via equation (6) which was solved using an efficient form of the Levinson algorithm. These coefficients were

augmented by  $128 - N_L$  zeros to allow the FFT routine to be used to efficiently evaluate the denominator of equation (5) at 128 equispaced points in wavenumber space for each frequency. The number of wavenumber bins corresponding to plane waves arriving from real arrival angles varies linearly from zero at d.c. to 128 at  $f_{\frac{1}{2}}$  (the frequency corresponding to the half wavelength of the array).

The region which does not correspond to real arrival directions is termed the 'non-physical' region of the frequency wavenumber spectrum and a full discussion and interpretation of this region is given at reference 5.

The power spectrum was scaled by the maximum value of  $\hat{S}_{ME}(f, k)$  and displayed in a waterfall format in figure 3 for  $N_L=16$  and figure 4 for  $N_L=24$ . (The 32 hydrophone array provides a maximum of 32 lags).

For comparison the Fourier estimate of the frequency wavenumber power spectrum was calculated as indicated in figure 5. After FFT'ing the acoustic data from each hydrophone to the same frequency resolution as for the MEM a second FFT was used to evaluate the spatial summation in equation (3) (see reference 5). Note that since only 32 hydrophones were analysed only 32 independent wavenumber bins are possible. At each frequency the spatial array of hydrophone data (ie  $\hat{X}_j(f)$  for  $j = 0, 1, \dots, 31$ ) was augmented with 96 zeros to allow the FFT routine to be used to efficiently evaluate the spatial Fourier series. This also allowed a ready comparison with the maximum entropy estimates since the same number of wavenumber points were estimated at each frequency for both methods.

Comparing figures 3 and 5 the increased wavenumber (and hence angular) resolution of the MEM is apparent. Comparing figures 3 and 4 where the number of spatial lags used has been increased from 16 to 24 respectively, two conclusions emerge:

- (1) The resolution has been improved by increasing the number of lags; and
- (2) At some frequencies (eg the lowest two frequencies of figure 4) and the MEM is starting to become unstable.

If the number of lags is increased further this instability becomes more pronounced.

## 6. CONCLUSIONS

The maximum entropy technique outlined in this paper is a computationally efficient way to increase the spatial resolution of estimates of the frequency wavenumber power spectrum of data from a linear array of sensors.

The effect of bias in the estimate of the spatial covariance function which is caused by windowing can cause misleading results. A simple technique for removing this bias has been shown to be effective when applying MEM to real data. The technique uses a subset of the available lags of the spatial covariance function; unfortunately it becomes unstable as the number of lags approaches the maximum possible (ie the number of hydrophones in the array). Further work using information theoretical ideas for determining the number of lags to be used is in progress.

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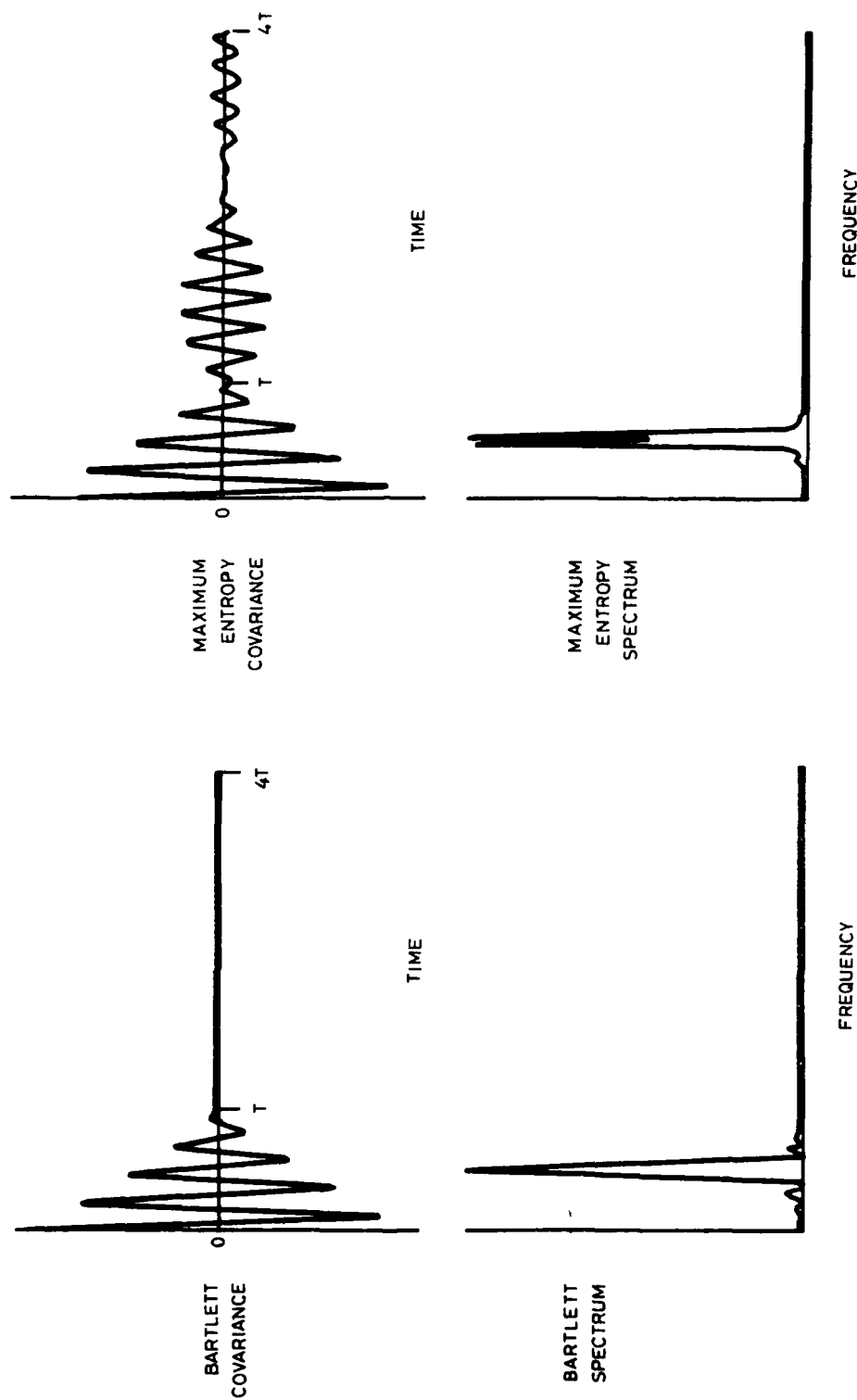


Figure 1. The effect of Bartlett windowing on the MEM



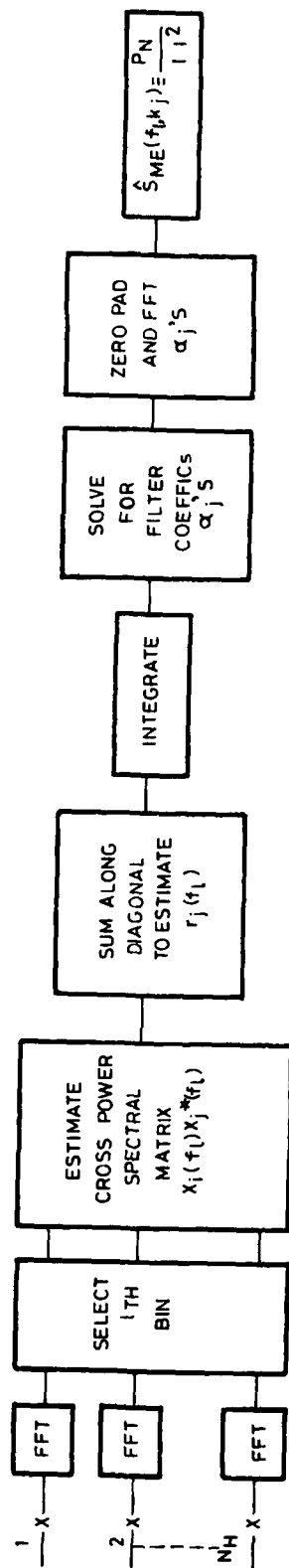


Figure 2. MEM estimation of wavenumber power spectrum



Figure 3. Maximum entropy estimates of frequency wavenumber power spectrum ( $N_L = 16$ )

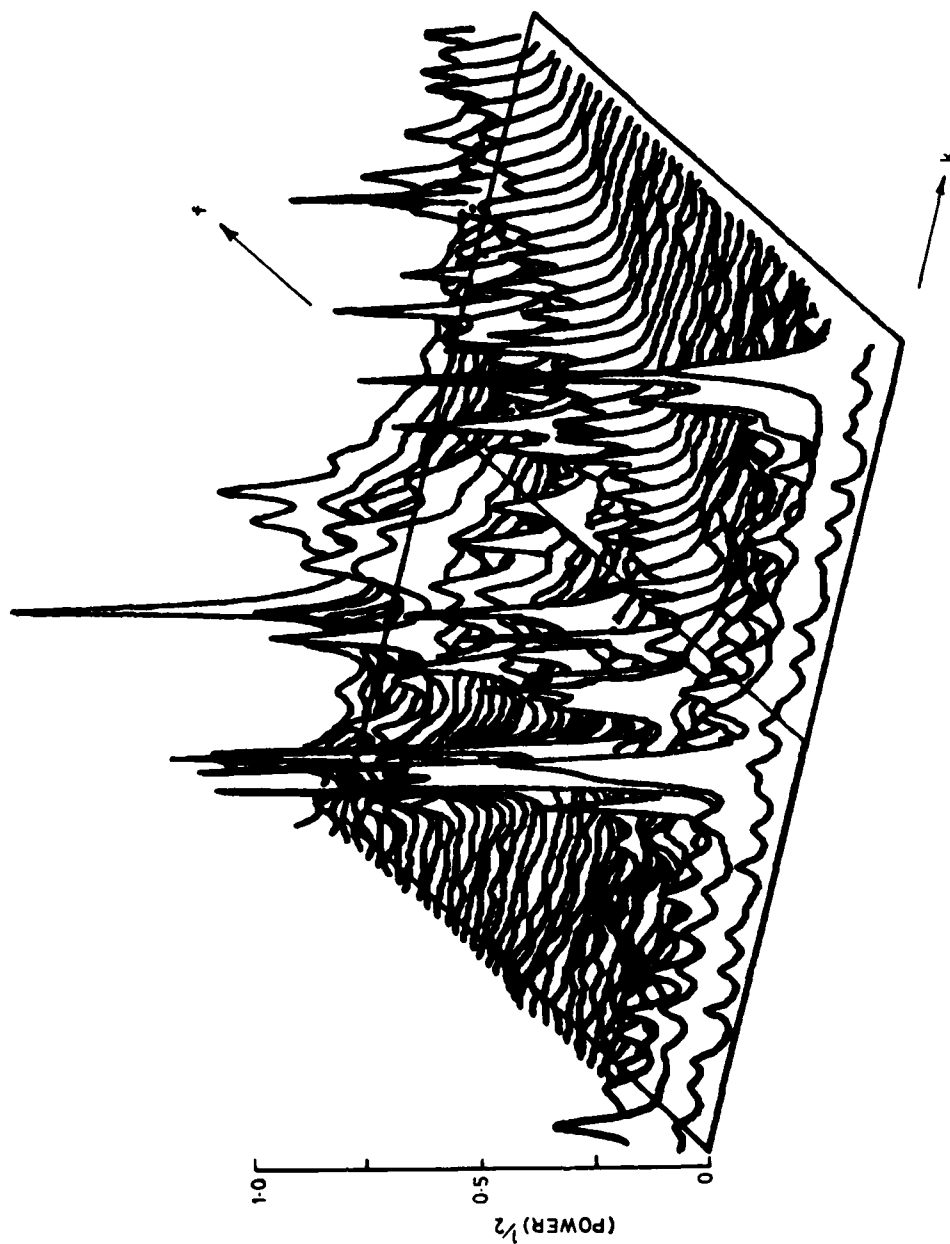


Figure 4. Maximum entropy estimates of frequency wavenumber power spectrum ( $N_L = 24$ )

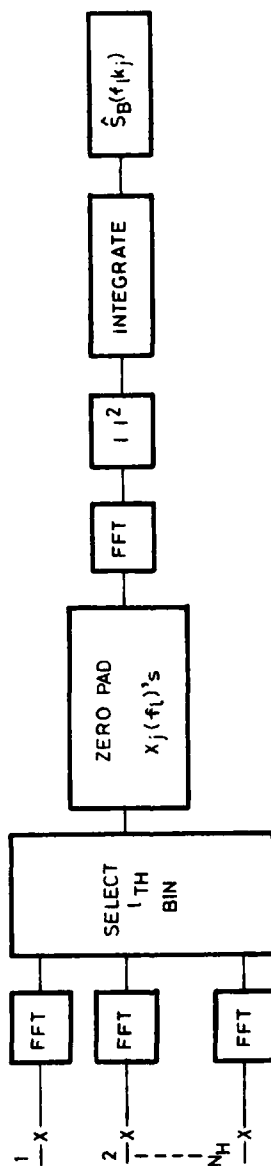


Figure 5. Fourier estimate of wavenumber power spectrum

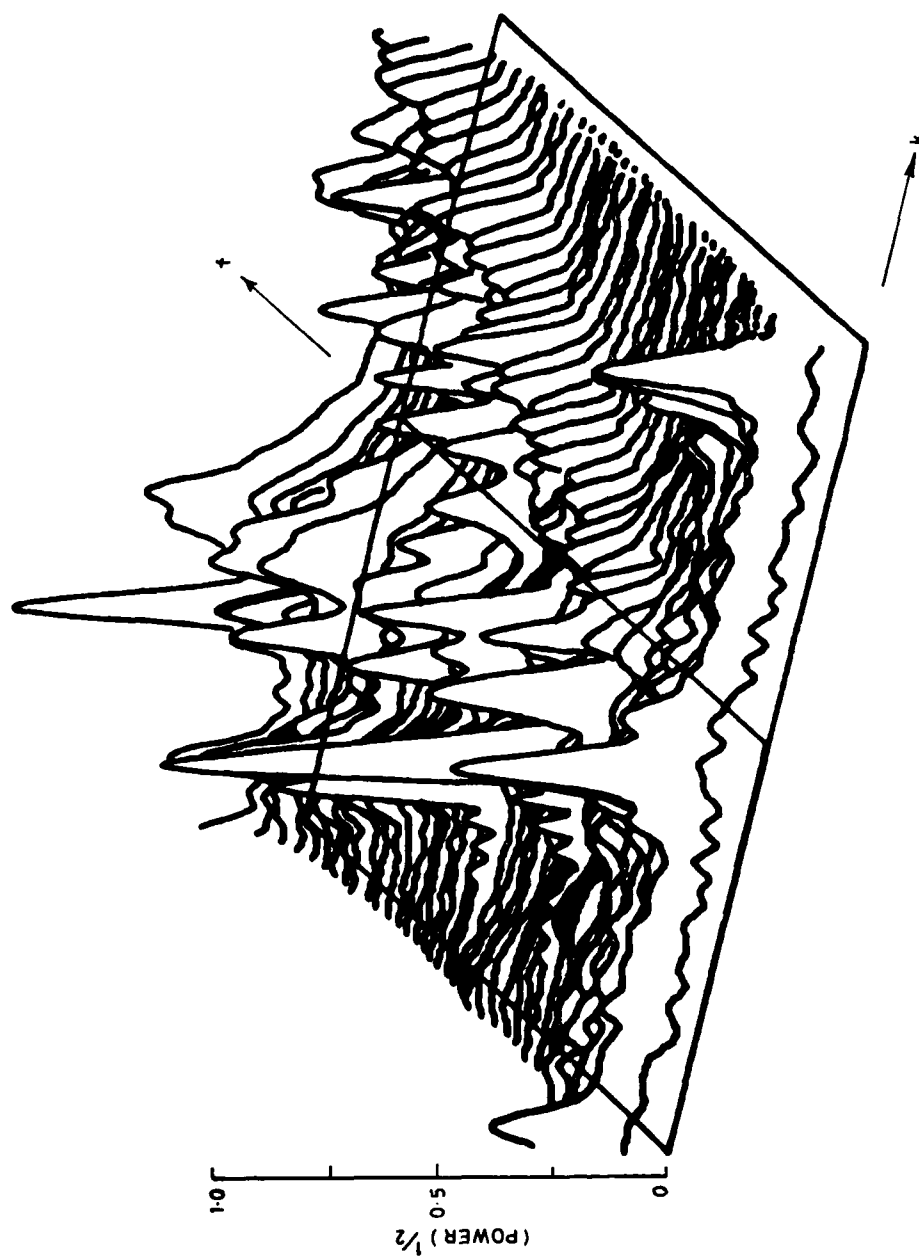


Figure 6. Bartlett (Fourier) estimates of frequency wavenumber power spectrum

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